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Surface pressures and their corrections for the flow past a finite-length plate in supersonic low density flow

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An experimental study of the flow past a thin finite length plate in a supersonic low density stream is reported. The paper discusses the corrections that are necessary for surface pressures measured under rarefied conditions. It is shown that the recent method of 'orifice' corrections due to Harbour & Bienkowski is versatile and reliable to use for both cold wall and insulated wall measurements. For the conditions of the experiment, the flow over the plate was found to be dominated by both leading-edge and trailing-edge interactions.

1. Introduction

The interaction of a hypersonic/supersonic laminar boundary layer and the essentially inviscid free stream under rarefied conditions has been the subject of many studies (Kendall 1957; Cheng *et al.* 1961; Oguchi 1963; Probstein & Pan 1963; Talbot 1963; Becker & Boylan 1967; McCroskey, Bogdonoff & Genchi 1967; Moulic & Maslach 1967; Shorenstein & Probstein 1968; Metcalf, Lillicrap & Berry 1969; Lillicrap & Berry 1970). Hypersonic strong and weak interaction theories (Hayes & Probstein 1959; Cheng *et al.* 1961; Oguchi 1963; Probstein & Pan 1963; Talbot 1963; Shorenstein & Probstein 1968) have been developed sufficiently to enable reasonable calculations of the surface pressure and heat transfer to be carried out. Experimental studies have verified (Kendall 1957; Becker & Boylan 1967; McCroskey *et al.* 1967; Moulic & Maslach 1967; Metcalf *et al.* 1969; Lillicrap & Berry 1970) the general characteristics of the theoretical model in these regions although detailed measurements for complete verification of the theoretical model in all the flow regimes is yet to be achieved. In particular, the leading-edge region of a sharp slender body and the termination effects – the so-called trailing-edge interaction – are still being explored both theoretically and experimentally (McCroskey *et al.* 1967; Metcalf *et al.* 1969; Lillicrap & Berry 1970).

For the purpose of the present paper, let it suffice to note that rarefied supersonic flows over flat plates generate a flow that is exclusively the result of viscous effects. The flow passes through a near free molecular region close to the leading edge followed by a merged layer region where the leading-edge shock wave and the boundary layer are merged together and the shock wave thickness too is appreciable. Downstream of the merged layer, the viscous layer and the shock wave have separated but still there is a strong coupling between the two. This is the so-called strong interaction region. This is followed by the weak interaction region where the coupling between the viscous layer and the outer shock layer is negligible to first order. When the free-stream Mach number is sufficiently low ($M_\infty \lesssim 6$), it has been observed (Moulic & Maslach 1967;

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Metcalf *et al.* 1969) that the strong interaction is usually absent and the flow passes straight from the merged to the weak interaction regime.

It is obvious, therefore, that in order to obtain a realistic experimental model of the flow of a rarefied supersonic flow over a slender body one needs to obtain reliable measurements of such quantities as heat transfer, shear stress and surface pressures (normal stress). Although a number of experimental measurements of surface pressure and heat transfer exist, the situation in regard to the surface pressure data is not entirely satisfactory as there appears to be wide disagreement between various investigations.

What is required in the wind-tunnel situation is the static pressure at the model surface. In practice there are connexions between the pressure sensor and the surface of the model on which one wants to know the pressure. In low density supersonic/hypersonic flows this means that the static pressure on the model wall is not, in general, equal to the 'measured' pressure because of non-continuum flow effects at the entrance to the pressure orifice. In particular, the measured pressure has to be corrected for the effects of temperature jump at the model surface and also for the shear stress at the surface. A number of well-known methods to perform these corrections exist (Kinslow & Arney 1967; Kinslow & Potter 1971; Metcalf, Berry & Dumbrel 1971; Smith & Lewis 1972; Harbour & Bienkowski 1973). These methods enable the static pressure to be calculated from the measured values in conjunction with other measured quantities such as the wall temperature, surface number density, heat transfer and shear stress.

2. Pressure measurements and their corrections

2.1. *The 'orifice' effect*

Surface pressure is measured either directly (say, with a flush-mounted transducer) or in a cavity inside the model connected to the surface via an orifice which is necessarily small if disturbances to the flow are to be minimum. An ideal orifice is the one whose depth is zero and whose diameter is much smaller than the local mean free path of the gas. A method of obtaining the true surface pressure p_w from such a cavity pressure has been proposed, for example, by Harbour & Bienkowski (1973). In practice, however, it is often difficult to obtain an orifice whose $d \ll \lambda_c$, where λ_c is the cavity mean free path, so that some account has to be taken of finite orifice diameter. Such a correction when d is not very much smaller than λ_c has also been proposed by Harbour & Bienkowski in the same paper. This scheme still has one drawback in that it assumes that the orifice has zero depth (or length), which again is not always attainable in practical situations.

2.2. *The 'tube' effect*

Recent studies (Metcalf *et al.* 1971; Smith & Lewis 1972; Kienappel 1971, 1974) have shown that not only the orifice geometry but also the connecting system can affect the 'measured' pressure significantly. Thus, when a small diameter tube is mounted with its one end flush with the model surface and the other connected to a relatively larger diameter tubing (which can effectively be considered as a cavity), then under rarefied flow conditions the measured pressure p_t can be substantially different from the surface pressure p_w. This is due, firstly, to the temperature difference between the gas

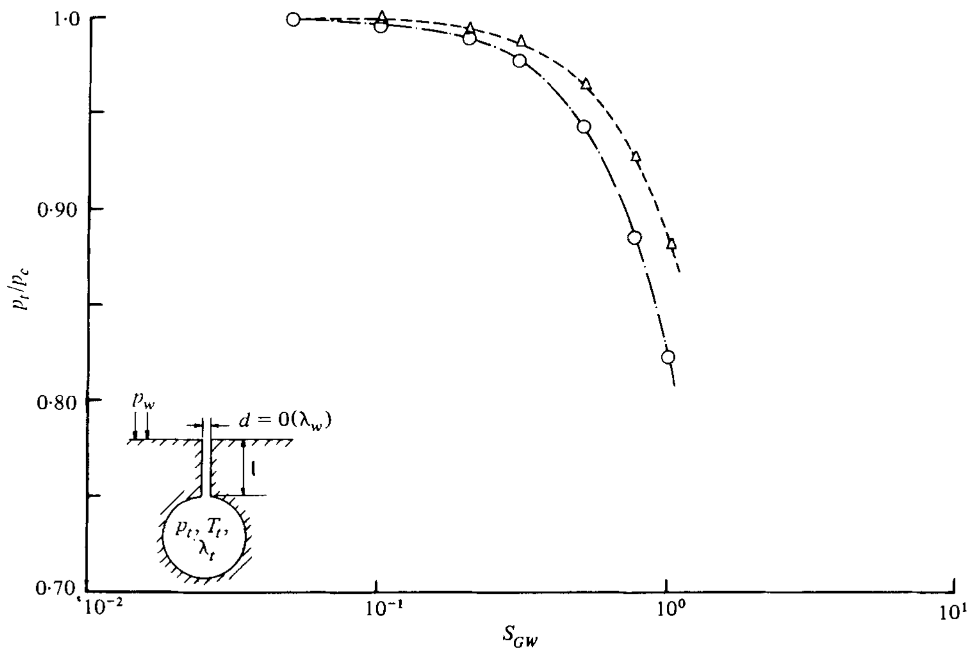

FIGURE 1. Variation of tube to cavity pressure ratio with respect to surface speed ratio. ○, $d/l \approx 0$; △, $d/l = 2{\cdot}0$.

next to the surface and the gas in the tube and cavity. Secondly, owing to slip effects, which would be present under these rarefied flow conditions, some of the molecules entering the tube may strike the walls of the tube and re-enter the stream without equilibrating in the cavity and hence these molecules will not contribute to the measured value p_t even though they would contribute to p_w, were the orifice absent. This would, then, render the pressures measured by the tube-orifice lower than those measured with an ideal orifice–cavity system. Based on free molecule flow assumptions, Hughes & DeLeeuw (1965) have shown that the measured pressure p_t is a function of incident speed ratio, length to diameter ratio of the tube and the inclination of the tube with respect to the flow. Figure 1 shows the dependence of p_t/p_c on the surface speed ratio S_{GW} for $d/l = 2$ (present model) and a very long tube orifice, both oriented perpendicular to the flow direction. It is clear that for small speed ratios the pressures measured by the tube orifices are very nearly the same as those by the cavity, but the differences become significant once the speed ratio becomes appreciable; that is, when there are strong rarefaction effects. This distinction between p_t and p_c was not made quite explicit in many of the earlier correction schemes. If, therefore, the tube orifice diameter is rendered smaller than the local mean free path and its inclination to the flow and the surface speed ratio are known, it is possible to correct the measured pressures for the 'tube' effect.

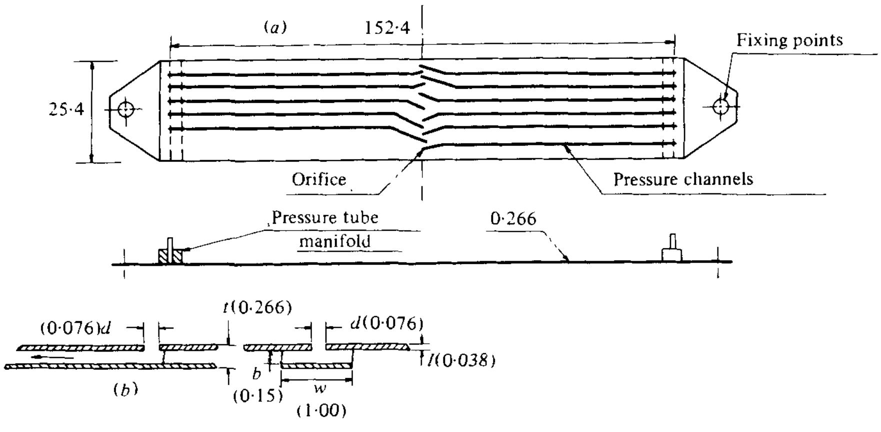

FIGURE 2. (a) Details of the experimental model. (b) Pressure passage details. All dimensions in mm.

Orifice locations

Orifice no.	Distance from leading edge (mm)
1	1·06
2	2·73
3	5·32
4	7·46
5	9·30
6	12·48
7	15·35
8	17·45
9	19·37
10	21·92
11	23·64

3. Experimental details

3.1. *The facility*

The experiments were conducted in an open jet continuous flow low density wind tunnel which provides supersonic flow through axisymmetric contoured nozzles. For the present tests the free-stream Mach number and pressure were 4·02 and 44 m Torr respectively. The stagnation temperature was approximately equal to the room temperature. Pressures were measured using a thermistor manometer with the bead being maintained at a constant temperature inside a water-jacketed cylinder.

3.2. *The model*

As stated in § 1, the aim was to study the flow field over a thin flat plate of finite length. This meant that the model had to be relatively free from both the leading-edge and trailing-edge thickness effects. The other two requirements were that the model had to be equipped with pressure orifices to enable surface pressures to be measured and to keep the wake region free from any extraneous disturbances such as pressure tubes or a sting arrangement.

The general layout of the model is shown in figure 2. It consists basically of two metal

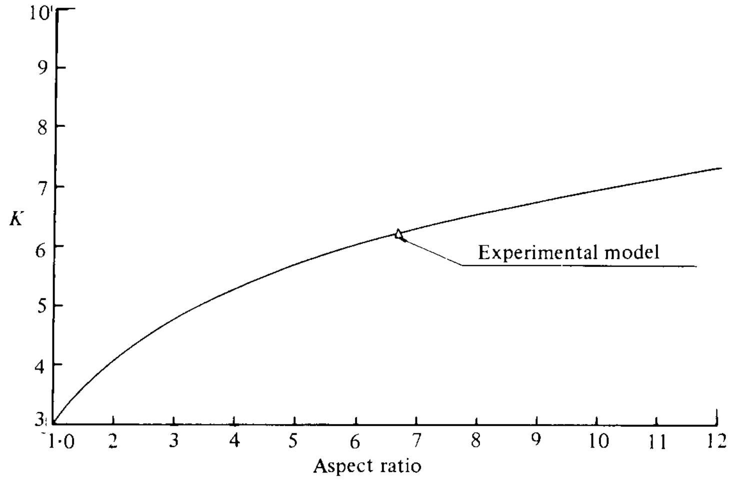

FIGURE 3. Variation of parameter K with aspect ratio of pressure passage.

skins stuck together. As seen from the figure, each surface tap has an orifice diameter of 0·076 mm and length about 0·038 mm. This is connected to a shallow (0·15 mm) but broad (1 mm) rectangular channel approximately 76 mm long which is etched into the metal skin. This channel is led into one of the two pressure manifolds either side of the model from where it is connected to the thermistor manometer. Although the pressure tap nearest to the leading edge was only 1·06 mm from it, the Reynolds number based on the leading-edge thickness was only of the order of 10, the thickness being of the order of two free-stream mean free paths. The leading-edge bluntness effects are therefore thought to have been not significant enough to affect the flow in the merged and weak interaction region.

Another important consideration in deciding the pressure orifice/lead line dimensions was the time response of the system. Since the pressure tap orifice is connected to a rectangular pressure passage, the time constant of the system could not be calculated using the relations such as those given by Schaaf & Cyr (1949). An expression for the time constant of a system connecting the orifice to the gauge *via* a rectangular pressure passage, based on the assumption of a free molecular flow, was, therefore, developed (Lloyd, private communication 1974). The expression is

$$t_e = \frac{3}{4(2\pi RT)^{\frac{1}{2}}}\left\{\frac{16\pi}{3}\cdot\frac{LV}{K}+\frac{8}{3}\cdot\frac{V}{r_0^2}\right\}, \tag{1}$$

where t_e = time constant; V = gauge volume; r_0 = radius of orifice; and K is a constant depending on the 'aspect ratio' of the passage and is given by

$$\frac{K}{wb^2} = a\ln\left\{\frac{(c+1)}{(c-1)}\right\} - \frac{2a}{3}(c-a) + \ln\left\{\frac{(c+a)}{(c-a)}\right\} - \frac{2}{3}\cdot\frac{(c-1)}{a}, \tag{2}$$

where $$c = \frac{(w^2+b^2)^{\frac{1}{2}}}{b} = (1+a^2)^{\frac{1}{2}} \quad \text{and} \quad a = w/b.$$

For a rectangular passage $w \gg b$, while for a square passage $w = b$.

Equation (1) shows that, for a given b and for given V, L and r_0, the larger the w, the smaller the time constant. Secondly with the increase in w, K too increases but rather slowly. Figure 3 shows K as a function of the aspect ratio of the passage. It should be noted that increase in w should not be so great as to invalidate the assumption that the volume of the tube is quite small compared to the gauge volume.

For the present model, $w = 1$ mm; $b = 0{\cdot}15$ mm; $L = 76$ mm; $r_0 = 0{\cdot}038$ mm; $V = 1{\cdot}934\,\mathrm{cm}^3$. The time constant was of the order of 20 s.

To the author's knowledge, this is the thinnest surface pressure tapped interference-free model ever to be used in low density flat plate measurements.

4. Measurements and application of correction procedure

The measurements were made at a free-stream Mach number of 4·02 and static pressure of 44 m Torr. The free-stream Reynolds number based on model chord was 1050 and the free-stream mean free path 0·144 mm. All the measurements were made with the model at zero incidence.

Surface pressure data with the above model and the flow field data using impact probe and hot wire measurements were obtained. The details of extracting the flow field information in a low Reynolds number viscous-dominated flow by combining pitot and hot wire data is given in Gai (1975). In the present paper, however, we shall concern ourselves only with the surface pressure measurements.

4.1. *The correction scheme*

It is clear from the discussion in § 2 that, for the present measurements, it is necessary to correct the pressures both for tube effect as well as orifice effect.

The measured pressures were thus corrected for the tube effect using the Hughes & DeLeeuw theory (1965) for $d/l = 2$ and orientated perpendicular to the flow. The surface speed ratio S_{GW} required for this purpose was obtained from the relation $S_{GW} = u_s/(2RT_{GW})^{\frac{1}{2}}$ and $T_{GW} \approx T_W$ assuming complete accommodation. The slip velocity u_s was obtained from the knowledge of the mean free path of the gas at the surface and the velocity gradient at the wall. This velocity gradient could be estimated from the velocity profile data obtained by combining the hot wire and impact probe measurements (see appendix for details).

Having obtained the cavity pressure p_c, these were then corrected for the orifice effect. The Harbour & Bienkowski (1973) procedure when $d = O(\lambda_c)$ was used. This scheme enables the calculation of surface pressure p_w from the cavity pressure p_c in terms of heat transfer $\dot{q}$ and the shear stress τ_w at the surface. It was first pointed out by the above authors that orifice corrections are, indeed, necessary even for adiabatic flow situations because of large shear effects especially in the leading-edge regions of a flat plate.

For zero heat transfer and unit normal and tangential accommodation coefficients, the expression of Harbour & Bienkowski (1973) simplifies to

$$p_w/p_c = \tfrac{1}{2}\{1+[1-D(1-\bar{P})]^{\frac{1}{2}}\}, \tag{3}$$

where
$$D = 2\pi\left(\frac{\gamma-1}{\gamma+1}\right)\left(\frac{\tau}{p_c}\right)^2$$

and
$$\bar{P} = f(d/\lambda_c, \gamma).$$

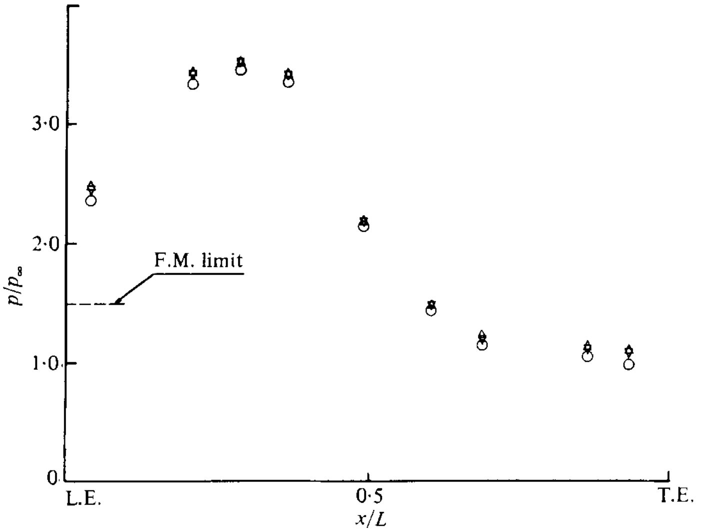

FIGURE 4. Surface pressures, measured and corrected data. ▽, uncorrected pressure; △, corrected for 'tube' effect; ○, corrected for 'orifice' effect.

From their static rig experiments, Kinslow & Arney (1967) have shown that the parameter $\overline{P}$ was a function only of d/λ_c for a given gas. Harbour & Bienkowski (1973) have made use of this result in their correction scheme.

As seen from equation (3), the orifice correction requires determination of D and $\overline{P}$. $\overline{P}$ can be calculated through the knowledge of the orifice diameter which is measured and the cavity mean free path λ_c. λ_c is known on the assumption that the wall and cavity temperatures are equal, which is reasonable for a very thin high thermal conductivity model and for $T_W \approx T_0$. Determination of D requires the knowledge of wall shear stress τ_w. In these experiments, this was rendered possible by knowing the velocity gradient at the wall $(\partial u/\partial y)_w$ at each measurement point. The velocity profile data obtained at all the pressure measurement stations was used for this purpose (see appendix). It must, however, be pointed out that the use of measured values of $(\partial u/\partial y)_w$ from the velocity profiles is susceptible to errors and this is one of the difficulties of the present approach. Estimation of the slip velocity in low density flat plate measurements has always presented difficulties and uncertainties (see, for example, Becker & Boylan 1967; Becker 1969). It is believed, nevertheless, that in the present instance these errors have not been very serious. Further, it may be pointed out that this is the first more direct approach for the determination of S_{GW} and τ_w than the ones adopted by either Harbour & Bienkowski (1973) (see discussion in appendix B – 'Comparison with Experiment' – of their paper) or Metcalf *et al.* (1971).

5. Discussion of results

Figure 4 shows the pressure measurements to which the above corrections have been applied. It is seen that both the corrections are small but orifice corrections are comparatively slightly larger. For example, at the first measurement point 1·06 mm from the leading edge, the correction due to the tube effect is about 1 % and that due to

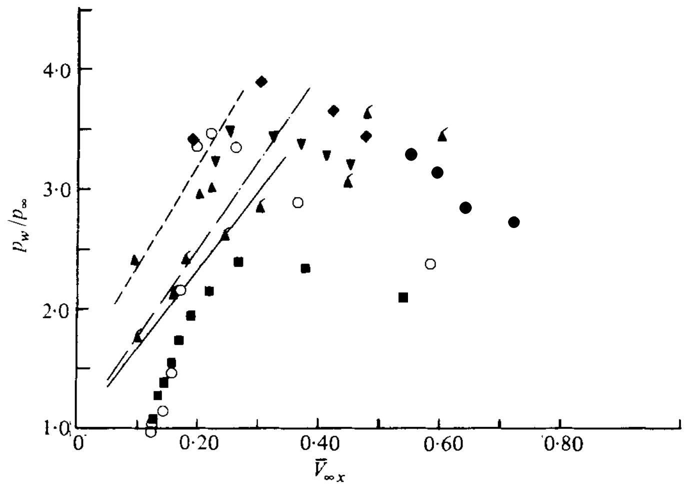

FIGURE 5. Corrected surface pressure *versus* rarefaction parameter. ◆, ▼, ▲, Moulic & Maslach (1967); ◭, Becker & Boylan (1967); ■, Metcalf, Lillicrap & Berry (1969); ○, present data. – – –, $O(1)$ weak interaction theory (Moulic & Maslach 1967); –·–, $O(1)$ weak interaction theory (Metcalf *et al.* 1969); ——, $O(1)$ weak interaction theory (present).

orifice effect is about 4 %. The same corrections for the rearmost measurement point are about 2 % and 10 % respectively. It will also be noted that in all cases the corrected pressure is less than the cavity pressure as well as the measured pressure.

In figure 5 the surface pressures are shown expressed in terms of the rarefaction parameter $\bar{V}_\infty$. It will be noted that the forwardmost point is in the fully merged layer while the plateau/peak is in the region of $\bar{V}_{\infty x}$ between 0·20 and 0·26. Thus, if one accepts that for the conditions of the present experiments the flow passes from the merged layer region straight into the weak interaction region, then this transition occurs at $\bar{V}_{\infty x} \approx 0{\cdot}20$. This result was further confirmed by flow field surveys by means of hot wire and impact probes.

Beginning of merging at $\bar{V}_{\infty x} \approx 0{\cdot}20$ is also consistent with the results obtained by various investigators. In general, various values have been quoted ranging from 0·15 to 0·25. In their cold wall experiments McCroskey *et al.* (1967), Becker & Boylan (1967) and Metcalf *et al.* (1969) all found that merging occurred when $\bar{V}_{\infty x} \approx 0{\cdot}15$. Moulic & Maslach (1967) in their insulated wall measurements on flat plates found that for merging $\bar{V}_{\infty x} \approx 0{\cdot}25$.

Figure 5 also shows results of the adiabatic wall data of Moulic & Maslach (1967) at $M_\infty \approx 6$ and of Becker & Boylan (1967) at $M_\infty \approx 4$. Both sets of data have been corrected for orifice effect. The $M_\infty \approx 6$ cold wall data of Metcalf *et al.* (1969) (corrected as per Harbour & Bienkowski (1973) procedure) is also shown on the same figure and it is seen that, while all the insulated wall data lie in the same region, the cold wall data of Metcalf *et al.* (1969) lie in a different region. It may also be noted that, while the insulated wall data in the weak interaction region of Moulic & Maslach (1967) and of Becker & Boylan (1967) show reasonable agreement with the first-order weak interaction theory, the present results show considerable deviation from the first-order theory. The same is true for the data of Metcalf *et al.* (1969).

There are very few data on flat plates at zero incidence showing the trailing-edge influence on surface pressures measured over the rear part of the plate. Only Metcalf *et al.* (1969) show the trailing-edge influence on flat plate measurements at zero incidence. The above authors also point.out that this trailing-edge influence is more pronounced at higher wall temperatures. The present measurements support this view (Vogenitz, Broadwell & Bird 1970; Gai 1975). It is seen from the present data that the flow over the plate is affected both by the leading-edge and trailing-edge interactions. First, over the front part of the plate, the pressure rises to form a peak and then decreases, which, as discussed earlier, indicates that in this region the flow is merged and of (leading edge) weak interaction type. Over the rear part of the plate, a strong trailing-edge interaction is evident from the fact that there is increasing divergence from the weak interaction theory. In fact all pressures subsequent to station 6 (i.e. the rear half) show definite trailing-edge influence.

6. Conclusions

The present investigation of a finite length flat plate at zero incidence in a Mach 4 low density flow has shown that:

(*a*) For proper interpretation of surface pressure data, account must be taken of both the tube and orifice effects. Of the methods available for orifice correction of surface pressures, the one proposed by Harbour & Bienkowski (1973) appears to be versatile and reliable as it can be used for correcting both cold wall and insulated wall data.

(*b*) Corrections are necessary for adiabatic models in rarefied flows. The present measurements thus confirm the assertion by Harbour & Bienkowski (1973) made earlier.

(*c*) The flow over the plate was influenced by both the leading-edge and trailing-edge interactions. Existence of the merged layer region was also confirmed, the merging occurring at $\bar{V}_{\infty x} \approx 0{\cdot}20$. This value is consistent with the findings of the previous investigators.

The work described above was carried out in the Aeronautical Engineering Department of the University of Bristol, and formed part of the Ph.D. thesis of the author. The author wishes to express his deep gratitude to his supervisor Mr D. H. Lloyd for his invaluable help and criticism throughout the course of this work. He would also like to record his sincere thanks to Mr S. C. Metcalf of RAE, Farnborough, for many helpful discussions and for his help towards the fabrication of the model.

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Appendix. Evaluation of S_{GW}, D, $\bar{P}$, (p_t/p_c) and (p_w/p_c)

(*a*) The surface speed ratio is given by

$$S_{GW} = \frac{u_s}{(2RT_{GW})^{\frac{1}{2}}},$$

where u_s = velocity of slip and T_{GW} = gas temperature at the wall.

Now, assuming that the wall temperature T_w, the gas temperature T_{GW} and the

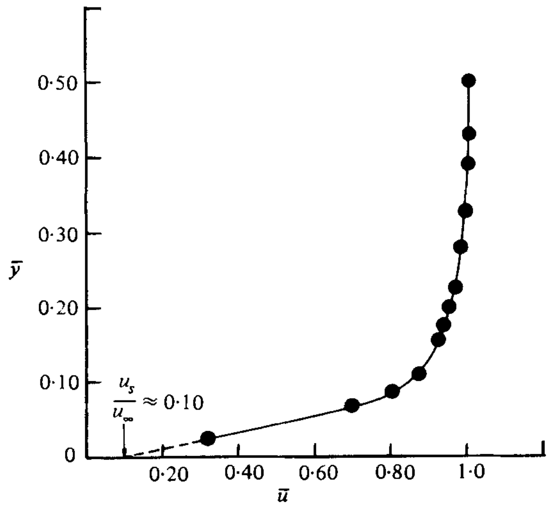

FIGURE 6. Typical velocity profile measured at $x/L = 0{\cdot}294$: $\partial\bar{u}/\partial\bar{y} \approx 9$.

stagnation temperature $T_{0\infty}$ are all approximately equal, which is reasonable for the present experimental conditions, we have

$$S_{GW} = \frac{u_s}{(2RT_{GW})^{\frac{1}{2}}} = \frac{u_s}{u_\infty}.M_\infty.\left(\frac{\gamma RT_\infty}{2RT_{GW}}\right)^{\frac{1}{2}} = \left(\frac{\gamma}{2}\right)^{\frac{1}{2}}.M_\infty.\left(\frac{T_\infty}{T_{0\infty}}\right)^{\frac{1}{2}}.\frac{u_s}{u_\infty}. \tag{A 1}$$

Further, $u_s = \xi\,\partial u/\partial y$, where ξ = slip coefficient. Assuming complete accommodation and Maxwellian distribution we can write

$$u_s = \lambda_w(\partial u/\partial y)_w, \tag{A 2}$$

where $\lambda_w = 2(\mu/\rho)\,[\pi/8RT_{GW}]^{\frac{1}{2}}$ = mean free path of the gas evaluated at T_{GW}. Then

$$\frac{u_s}{u_\infty} = \frac{\lambda_w}{L}\left\{\frac{\partial(u/u_\infty)}{\partial(y/L)}\right\}_w = \frac{\lambda_w}{L}\left(\frac{\partial\bar{u}}{\partial\bar{y}}\right)_w, \tag{A 3}$$

where u_∞ is the velocity in the free stream and L is the length of the plate.

(*b*) To evaluate the parameter D in the Harbour & Bienkowski (1973) correction scheme, we need to know the wall shear stress τ_w. D is given by

$$D = [2\pi(\gamma-1)/(\gamma+1)]\,(\tau_w/p_c)^2,$$

where τ_w = wall shear stress and p_c = cavity pressure.

We may write D as

$$D = 2\pi\frac{\gamma-1}{\gamma+1}\left[\mu_w\left(\frac{\partial\bar{u}}{\partial\bar{y}}\right)_w.\frac{u_\infty}{L}.\frac{1}{p_c}\right]^2, \tag{A 4}$$

where μ_w is evaluated at T_w.

(*c*) As a typical numerical calculation, consider the velocity profile data shown in figure 6. This was obtained at $x/L = 0{\cdot}294$, where the measured value of p/p_∞ was 3·522.

From the velocity profile it is seen that the measured value of $(\partial\bar{u}/\partial\bar{y})_w \approx 9$. The calculated value of λ_w/L is 0·124, giving $u_s/u_\infty = 0{\cdot}112$. The corresponding S_{GW} is then 0·184 so that $p_t/p_c = 0{\cdot}996$ at this point. This, in turn, gives $p_c/p_\infty = 3{\cdot}536$, which shows that the cavity pressure is slightly more than the pressure recorded by the tube orifice consistent with Hughes & DeLeeuw (1965) theory. It may also be noted that while direct extrapolation to the wall, from figure 6, gives $u_s/u_\infty \approx 0{\cdot}1$, calculation gives $u_s/u_\infty = 0{\cdot}112$ which is very reasonable.

The next step is to calculate p_w/p_c, which is the true pressure required. Again, using $(\partial\bar{u}/\partial\bar{y})_w$ and all other known values, D is computed from equation (A 4). For the present example, $D = 0{\cdot}0783$.

For $\bar{P}$ we need λ_c which can, to a sufficient degree of accuracy, be taken as equal to λ_w calculated above, so that in the present instance the parameter $d/\lambda_c = 0{\cdot}24$, giving $\bar{P} = 0{\cdot}11$.

Using these data, p_w/p_c is finally calculated and in the present case $p_w/p_c = 0{\cdot}982$, giving the true pressure as $p_w/p_\infty = 3{\cdot}473$.

REFERENCES

Becker, M. & Boylan, D. E. 1967 In *Rarefied Gas Dynamics*, vol. 2 (ed. C. L. Brundin), p. 993. Academic Press.

Becker, M. 1969 In *Rarefied Gas Dynamics*, vol. 1 (ed. L. Trilling & H. Y. Wachman), p. 515. Academic Press.

Cheng, H. K., Hall, J. G., Gollian, T. C. & Hertzberg, A. 1961 *J. Aero. Sci.* **28**, 353.

Gai, S. L. 1975 Ph.D. dissertation, University of Bristol.

Harbour, P. J. & Bienkowski, G. K. 1973 *Phys. Fluids* **16**, 600.

Hayes, W. D. & Probstein, R. F. 1959 *Hypersonic Fow Theory*, chap. 9. Academic Press.

Hughes, P. C. & Deleeuw, J. H. 1965 In *Rarefied Gas Dynamics*, vol. 1 (ed. J. H. Deleeuw), p. 653. Academic Press.

Kendall, J. 1957 *J. Aero. Sci.* **24**, 47.

Kienappel, K. 1971 *A.I.A.A. J.* **9**, 749.

Kienappel, K. 1974 *Z. Flugwiss.* **22**, 240.

Kinslow, M. & Arney, G. D. 1967 *AGARDOgraph* 119.

Kinslow, M. & Potter, J. L. 1971 In *Rarefied Gas Dynamics* (ed. D. Dini, C. Cercignani, S. Nicolls & L. Trilling). To be published by the Academic Press.

Lillicrap, D. C. & Berry, C. J. 1970 *Phys. Fluids* **13**, 1146.

McCroskey, W. J., Bogdonoff, S. M. & Genchi, A. P. 1967 In *Rarefied Gas Dynamics*, vol. 2 (ed. C. L. Brundin), p. 1047. Academic Press.

Metcalf, S. C., Lillicrap, D. C. & Berry, C. J. 1969 In *Rarefied Gas Dynamics*, vol. 1 (ed. L. Trilling & H. Y. Wachman), p. 619. Academic Press.

Metcalf, S. C., Berry, C. J. & Dumbrel, S. G. 1971 In *Rarefied Gas Dynamics* (ed. D. Dini, C. Cercignani, S. Nicolla and L. Trilling). To be published by the Academic Press.

Moulic, E. S. & Maslach, G. J. 1967 In *Rarefied Gas Dynamics*, vol. 2 (ed. C. L. Brundin), p. 971. Academic Press.

Oguchi, H. 1963 In *Rarefied Gas Dynamics*, vol. 2 (ed. J. Laurmann), p. 191. Academic Press.

Probstein, R. F. & Pan, Y. S. 1963 In *Rarefied Gas Dynamics*, vol. 2 (ed. J. Laurmann), p. 194. Academic Press.

Schaaf, S. & Cyr, J. 1949 *J. Appl. Phys.* **20**, 860.

Shorenstein, M. L. & Probstein, R. F. 1968 *A.I.A.A. J.* **6**, 1896.

Smith, J. A. & Lewis, J. H. 1972 *A.I.A.A. J.* **10**, 1183.

Talbot, L. 1963 *A.I.A.A. J.* **1**, 1169.

Vogenitz, F. W., Broadwell, J. E. & Bird, G. A. 1970 *A.I.A.A. J.* **7**, 1099.